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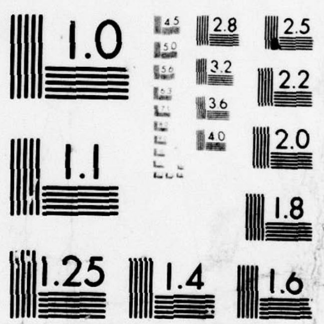

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MODELING PILOT RESPONSE DELAYS TO BEACON COLLISION AVOIDANCE SYSTEM COMMANDS

B. Billmann

T. Morgan

J. Windle

OCTOBER 1979

FINAL REPORT

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16. Abstract

This report contains the results of an analysis of pilot response delays to collision avoidance commands displayed in the cockpit of a General Aviation Trainer (GAT) simulator. Data were obtained from previous tests conducted in the Beacon Collision Avoidance/GAT simulation at the National Aviation Facilities Experimental Center (NAFEC), Atlantic City, New Jersey. Subjects were general aviation pilots with a wide range of experience. Statistical curve fitting techniques are applied to response delay data. For fixed geometries, velocities, and aircraft response rates, the separation at the point of closest approach between aircraft responding to collision avoidance system (CAS) commands is inversely related to the length of pilot delay in responding to the CAS command. In experimental simulations used to measure that collision avoidance system effectiveness, the modeling of pilot response delays should be as realistic as as is practicable. The results of this study provide a more realistic model than previously available. The Gamma distribution was the best distribution applicable to the queuing processes which are conceptually similar to the process that is generating the pilot response delay times. In terms of minimum error mean square, lack of bias, and uniformity of fit, the use of the Gamma distribution was found to be superior in approximating the empirical data. The recommendation to use the Gamma distribution in modeling pilot response delays in subsequent experimentation at NAFEC is made.

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Pilot Response Delay
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METRIC CONVERSION FACTORS

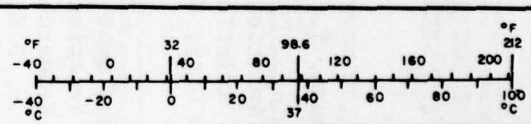
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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INTRODUCTION

PURPOSE.

This document describes the results of research conducted to investigate the generating mechanism behind the distribution of pilot response delays; specifically, delays in response to beacon collision avoidance system (BCAS) commands. The analysis centers on identifying the theoretical distribution that most closely models the empirical pilot response delay time data. The investigation was undertaken at the suggestion of the BCAS Logic Working Group. The report provides the necessary information to allow all BCAS logic fast-time and real-time studies to use the same model of the pilot response delays during research in support of the BCAS effectiveness analysis.

BACKGROUND.

Currently, several efforts are in progress to analyze the effectiveness of various aspects of BCAS logic. These studies have been conducted using different models for the distribution of pilot response delay times. During July - September 1977, the National Aviation Facilities Experimental Center (NAFEC) conducted an experiment to measure pilot assessment of certain aspects of BCAS (references 1 and 2). These data were used to examine the applicability of different pilot response models.

METHOD.

The general aviation trainer (GAT 2A) is a flight simulator that models a medium piston-powered twin-engine aircraft. Peripheral equipment used to drive the computer-generated projection of intruders is shown in figure 1. Data collected during the pilot GAT 2A/BCAS interaction experiment were used as a data base for this analysis. Data were collected while the pilots were flying the GAT 2A under simulated instrument conditions. During these flights, visual targets were presented along with the BCAS-generated commands. Several types of commands were presented; however, the analysis in this report is concerned only with the delay in pilot responses to the four positive BCAS commands; turn left, turn right, climb, and descend. Figure 2 depicts the cockpit display devices used to present BCAS commands to the pilots.

The distributions currently being used to model the pilot response delay to BCAS commands have a 6-to-7-second mean. The shapes of the distributions vary widely, ranging from the Uniform, with a range of 4.5 seconds to 7.5 seconds, to a Normal (7, 1) distribution truncated at 1 and 9 seconds. However, none of these models possess the heavy right tail that represents the significant proportion of late responses that occurred. More meaningful results in experimental simulation can be obtained if the model of the pilot response to collision avoidance commands included this characteristic.

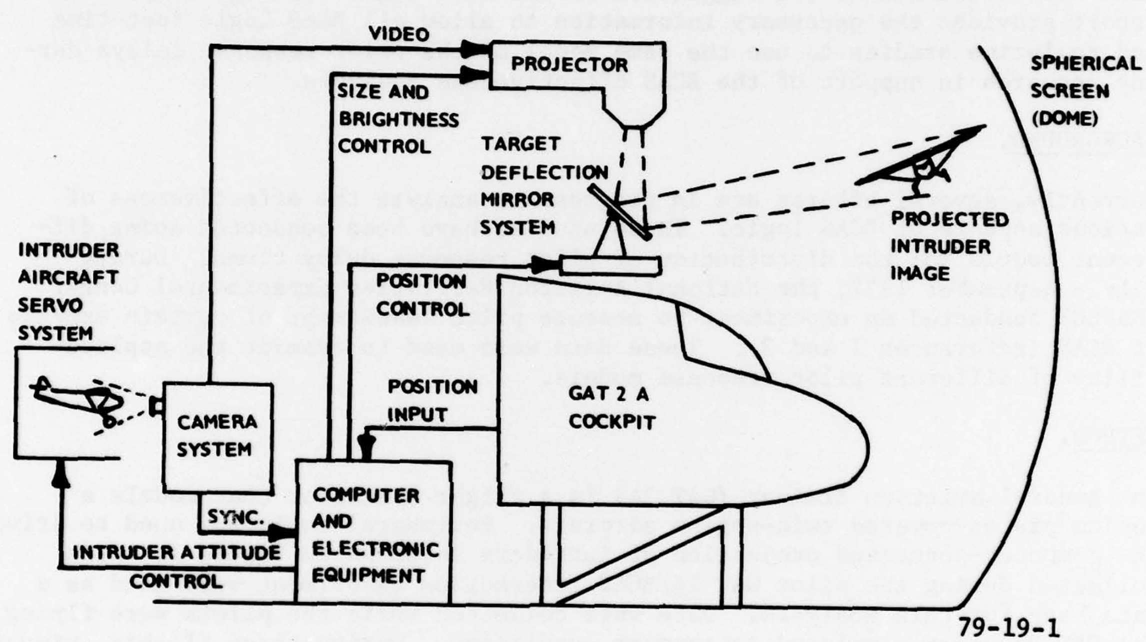
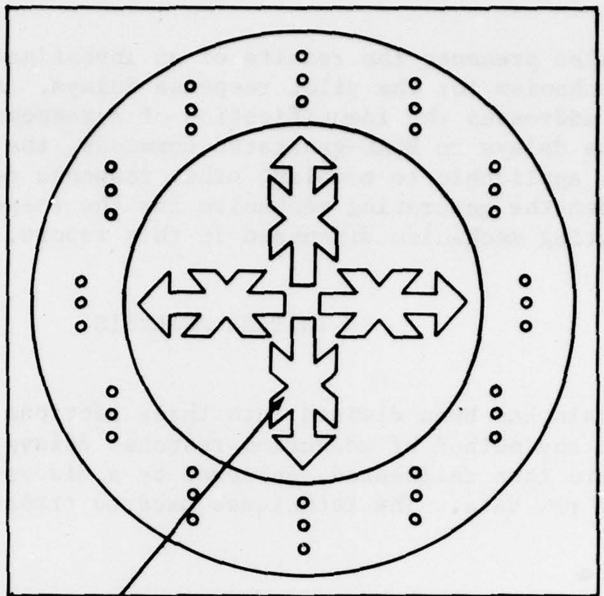
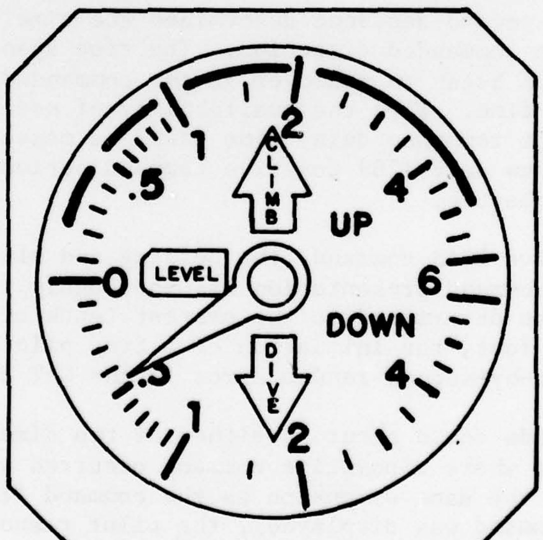


FIGURE 1. GAT 2A PROJECTION SYSTEM



BADCOM DISPLAY

A POSITIVE COMMAND IS PRESENTED BY ILLUMINATING A GREEN ARROW IN THE PROPER DIRECTION



IVSI DISPLAY 79-19-2

FIGURE 2. BCAS COCKPIT DISPLAY DEVICES

This report also presents the results of an investigation of the underlying generating mechanism for the pilot response delays. Although this study specifically addresses the identification of a response model associated with pilot response delays to BCAS-generated commands, the use of the Gamma distribution may be applicable to modeling other response delays. This would be true especially when the generating mechanism for the response delays is similar to the generating mechanism discussed in this report.

INITIAL ANALYSIS

Initial analysis has been divided into three sections. The data collection technique and the method of measuring response delays are presented. Basic assumptions are then delineated, followed by a discussion for the necessity to transform the raw data. The techniques used to transform the data are reviewed.

DATA SOURCE.

The standard Air Traffic Control Simulation Facility (ATCSF) data reduction and analysis (DR&A) program (reference 3) in conjunction with Cal-Comp plots of all encounters was used to identify the time at which a positive BCAS command was first presented to a pilot. The second-by-second printout from the ATCSF DR&A program provided the true heading and altitude of the GAT 2A during the time periods in which positive commands were displayed to the pilot. This second-by-second sequence determined the time that the GAT 2A began its maneuver in the commanded direction. The time span from command presentation until the GAT 2A began to maneuver in the commanded direction is defined as response delay time. With the availability of second-by-second data, each individual pilot response delay time could be measured to the nearest second. The amount of raw data (289 positive command periods) justified the use of this measurement procedure.

For each positive BCAS command, the heading and altitude of the GAT 2A during the period of command presentation was thoroughly examined. Since the GAT 2A heading could be determined to the nearest tenth of a degree and the altitude to the nearest foot, the initiation of a true pilot response could be separated from the second-by-second random error in the GAT 2A heading.

Positive commands could occur in either of two dimensions, horizontal or vertical. For cases where a positive command occurred when an aircraft was already maneuvering in the same dimension as the command (i.e., horizontal maneuvering when a turn command was displayed), the pilot response delay data were not collected. The removal of these data points helped eliminate any effect the aircraft aerodynamics would have on pilot response delay times.

If no pilot response was detected prior to the termination of the BCAS command, no pilot response delay data could be collected for the command period. Throughout the experimentation, 289 positive command periods occurred. Using the above procedures, 264 pilot response delays were measured.

BASIC ASSUMPTIONS.

Further investigation into the shape of the distribution of the pilot response delay times requires that several nonrestrictive basic assumptions be made. Since response data were not collected when no pilot response was detected, the distribution obtained is actually the conditional distribution of pilot response delay times given that the command length equaled or exceeded the response delay.

The observed distributions of the pilot response delay times were tabulated for the eight combinations of the four possible command directions and two BCAS logic sensitivity levels (amount of warning time provided to the pilot before point of closest approach). The differences in the sample means were tested using a multiple comparisons T test. No significant differences were found. An approximate chi-square test did not detect differences in the sample variances of the response delays due to sensitivity level or command direction. This result supports the assumption that the distribution of pilot response delays are independent of command direction and BCAS logic sensitivity. This independence allowed the pooling of data to form a single data base. Results of the approximate chi-square test and of the pooling strategy are reviewed in appendix A. Figure 3 presents the histogram of the pooled raw data points.

DATA TRANSFORMATION.

The raw data presented in figure 3 required two modifications prior to beginning any curve-fitting process. The first modification involved the identification and elimination of excessively large response delays from the data base. The second modification transformed the raw data to eliminate the bias caused by the command length distribution. Since data were only collected when the length of the command presentation period exceeded the pilot response delay, the resulting data base was biased by the distribution of the lengths of command periods.

LARGE RESPONSE DELAYS. The original data base contained observations which ranged from 1 second to 48 seconds. The collision avoidance system being tested generated commands 15 to 25 seconds prior to the closest point of approach. Analysis indicated that with response delays of 15 seconds or more, the achieved separations were about the same as what would have resulted with no response at all. The original data set of 264 observations included 11 observations where the response delays were 15 seconds or greater. ($P\{\text{delay} \geq 15\} = 0.042$). Instead of trying to fit the 11 points, which in essence represented no response, the fit was conducted in the range of 1 to 14 seconds. The 11 observations in question represent the probability of no response which can be modeled separately from the modeling of response delays.

UNBIASING TECHNIQUES.

As stated before, the observed distribution is the conditional distribution of pilot response delays given that the command length is at least as long as the response delay. The observed data do not include some responses that would

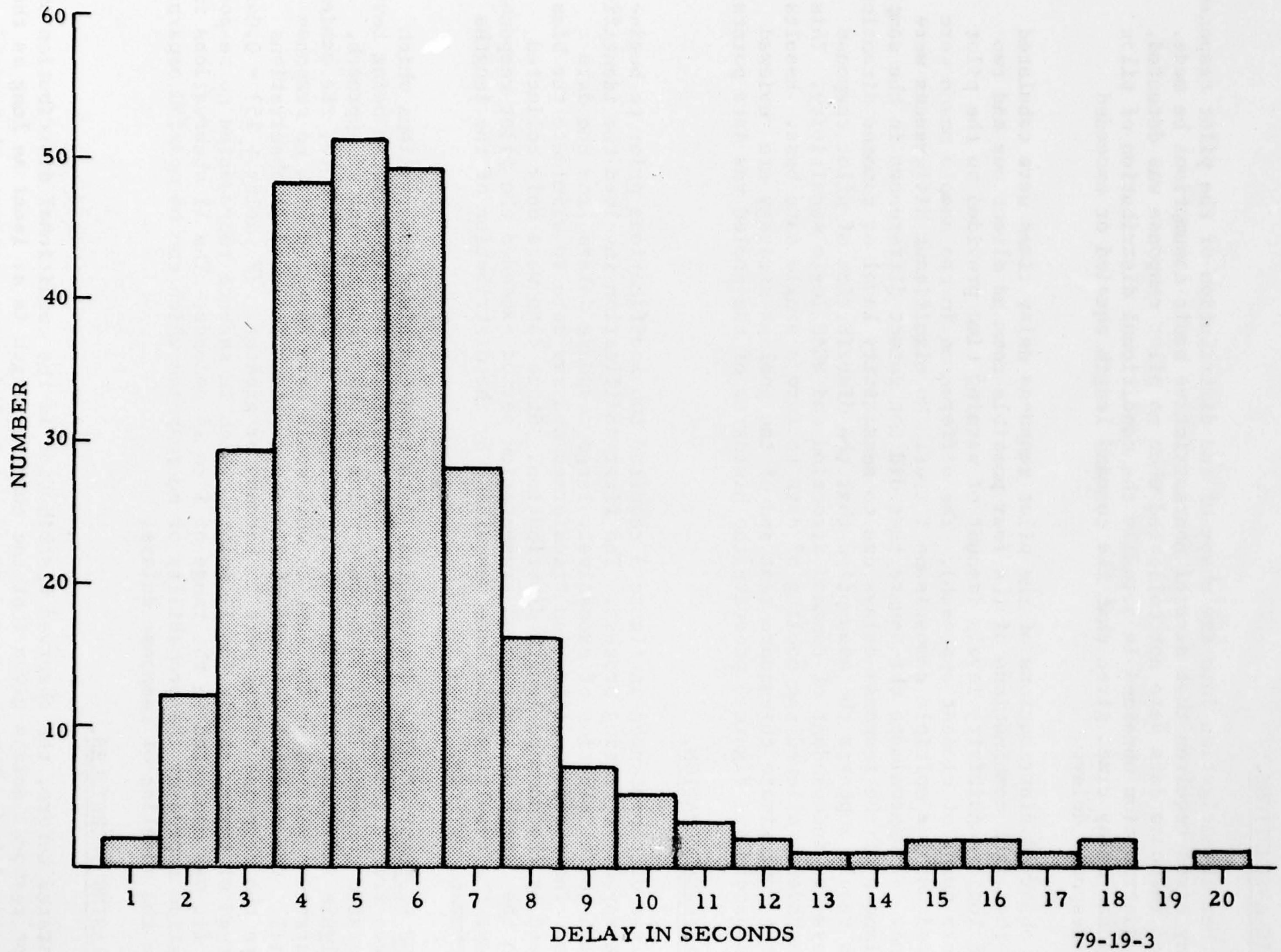


FIGURE 3. HISTOGRAM OF RAW DATA POINTS

have occurred had some BCAS commands not terminated prior to 14 seconds duration. Therefore, some data have been censored by the sampling scheme; i.e., the cases where pilot response delays exceeded the command length. This fact indicates that the observed distribution has been biased by the censoring of some response delays that would have occurred had all command lengths been at least 14 seconds long. To correct for the bias, let the events;

A = the occurrence of a command length equaling or exceeding the response delay

and

B_k = the occurrence of a response delay of k seconds, k = 1,2, .., 14

Note the sets B_k are mutually exclusive. Bayes Formula (reference 4) states:

$$P(B_k) = \frac{P(B_k|A)}{P(A|B_k)} \sum_{j=1}^{n} P(A|B_j)P(B_j)$$

Now, $P(B_k \mid A)$ is the probability of observing a response delay of k seconds given that the command length equals or exceeds the response delay. These probabilities are obtained directly from the distribution of observed pilot response delays.

$P(A \mid B_k)$ is the probability that the command length equals or exceeds the the response delay given the response delay was k seconds. This is the unconditioned probability that a command length equals or exceeds k seconds. Since all positive BCAS command displays must equal or exceed 4 seconds, $P(A \mid B_k) = 1.0$ for k = 1, 2, 3, 4.

Additionally, the BCAS display is updated once every 2 seconds; hence, possible commands lengths are 4, 6, 8, 10, 12, 14, . . . seconds.

This fact implies that

$P(A|B_{k-1}) = P(A \mid B_k)$ for k = 6, 8, 10, 12, 14.

Values for these probabilities were obtained from the empirical distribution of command lengths.

Although $\sum_{j=1}^{n} P(A|B_j)\ P(B_j)$ is unknown, that sum is independent of k, and the fact that $P(B_k)$ is a probability distribution can be used to obtain unbiased values of $P(B_k)$.

Since
$$\sum_{k=1}^{n} P(B_k) = \sum_{j=1}^{n} P(A|B_j)P(B_j) \sum_{k=1}^{n} P(B_k|A)/P(A|B_k) = 1$$

then
$$\sum_{j=1}^{n} P(A|B_j)P(B_j) = \left(\sum_{k=1}^{n} P(B_k|A)/P(A|B_k)\right)^{-1}$$

and $P(B_k) = \left(\sum_{k=1}^{n} P(B_k|A)/P(A|B_k) \right)^{-1} P(B_k|A)/P(A|B_k)$

The effect of the unbiasing procedure on the original data is reviewed in table 1.

The required evaluation of $\left(\sum_{k=1}^{n} P(B_k|A)/P(A|B_k) \right)^{-1} = (1.0855)^{-1} = 0.0912$

is obtained from table 1. The resulting unbiased empirical data set $\{\underset{\sim}{X}_i\}_{i=1}^{14}$ is used as the data base for all curve-fitting procedures that follow. The plot representing the distribution of the transformed data is shown in figure 4.

CURRENT MODELS

Two models are currently being used for the distribution of pilot response delays. The distribution in use at the ATCSF at NAFEC is the truncated Normal with a mean of 7 seconds and a standard deviation of 1 second. Upper and lower truncation points of 1 and 9 seconds are used to limit the range of the distribution. Mitre, Inc., is using a Uniform distribution with the range 4.5, 7.5. In this section, the empirical data are fitted to several members of both the Normal and Uniform families of distributions.

THE NORMAL FAMILY.

Three curves from the Normal family were compared with the empirical distribution of the transformed data. The three curves compared are the best-fit nonlinear least squares Normal curve, the Normal curve generated by the sample moments of the empirical data, and the ATCSF model of pilot response delay.

BEST NONLINEAR LEAST SQUARES NORMAL FIT. The BMDO7R program (reference 5) was used to fit the data. BMDO7R uses the Gauss-Newton iterative procedure to estimate the distribution parameters. The system converged quite rapidly. Using this program, a mean of 5.18 seconds and a variance of 3.72 seconds squared were obtained.

The analysis of the fit to the 14 data points in table 1 is good; however, several problems are evident as shown on figure 5. Comparing the unbiased data sample mean, 5.57 seconds, with the fitted mean, 5.18 seconds, shows that the fit underestimates the data mean by 7 percent and the fitted variance, 3.72 seconds squared, underestimates the sample variance, 4.80 seconds, by 20 percent.

The symmetric property of the Normal causes an increasing underestimation of the empirical data for all response delays ≥ 8 seconds. The normal distribution cannot fit the tails of the unbiased empirical distribution. The estimated mean square error (EMS) of the fit is 8.4×10^{-5}.

TABLE 1. EFFECT OF UNBIASING PROCEDURE

Length of Delay	$P(B_k \mid A)$ (Outliers Removed)	$P(A \mid B_k)$	$P(B_k \mid A)/P(A \mid B_k)$	$P(B_k) = \underset{\sim}{X}_k$ (Unbiased Empirical Distribution)
1	0.0079	1.0	0.0079	0.0073
2	0.0474	1.0	0.0474	0.0437
3	0.1146	1.0	0.1146	0.1056
4	0.1897	1.0	0.1897	0.1747
5	0.2016	0.9328	0.2161	0.1991
6	0.1937	0.9328	0.2076	0.1912
7	0.1107	0.8379	0.1321	0.1217
8	0.0632	0.8379	0.0754	0.0695
9	0.0277	0.8103	0.0342	0.0315
10	0.0198	0.8103	0.0244	0.0225
11	0.0119	0.7826	0.0152	0.0140
12	0.0079	0.7826	0.0101	0.0093
13	0.0040	0.7352	0.0054	0.0050
14	0.0040	0.7352	0.0054	0.0050

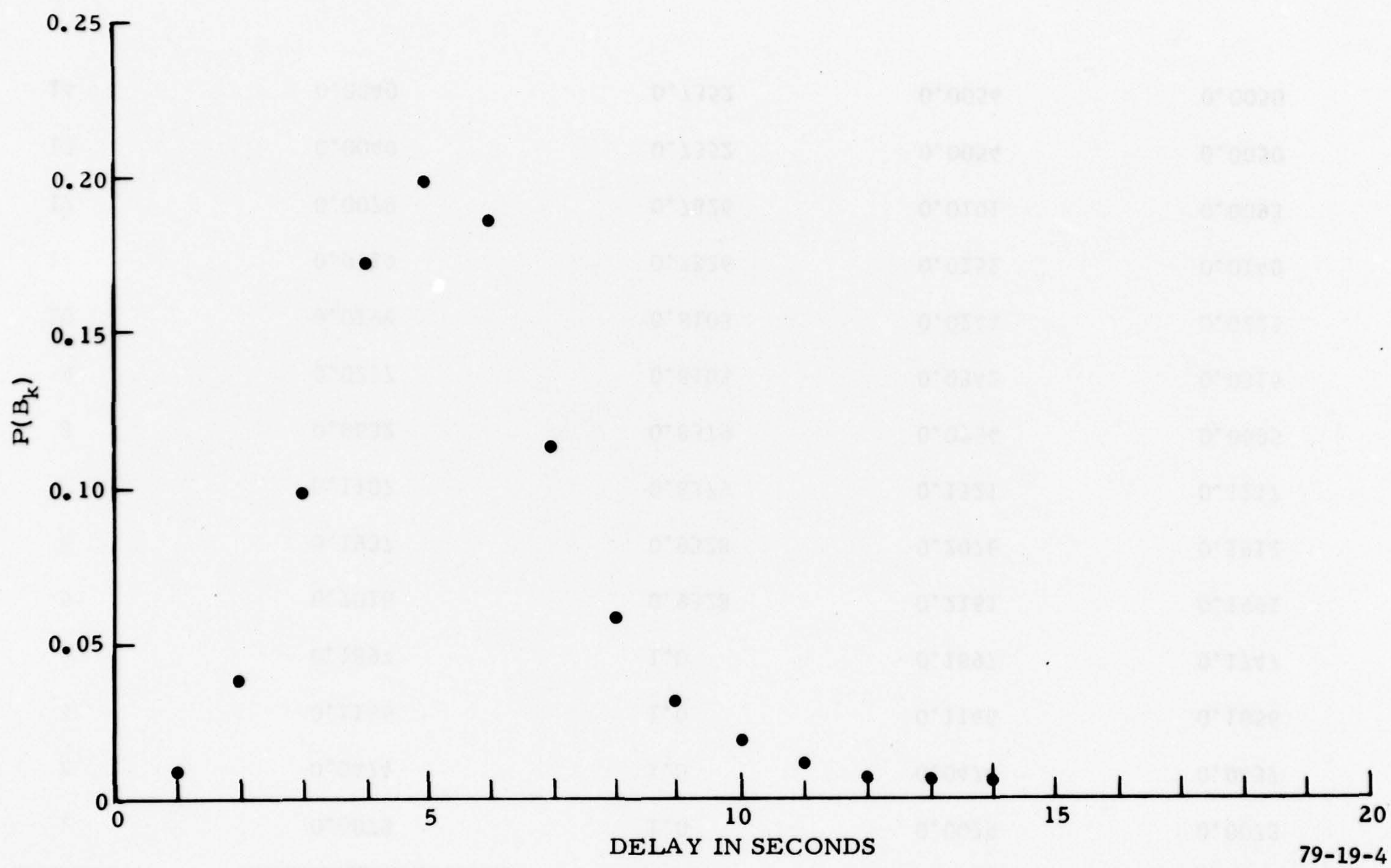


FIGURE 4. UNBIASED EMPIRICAL DISTRIBUTION

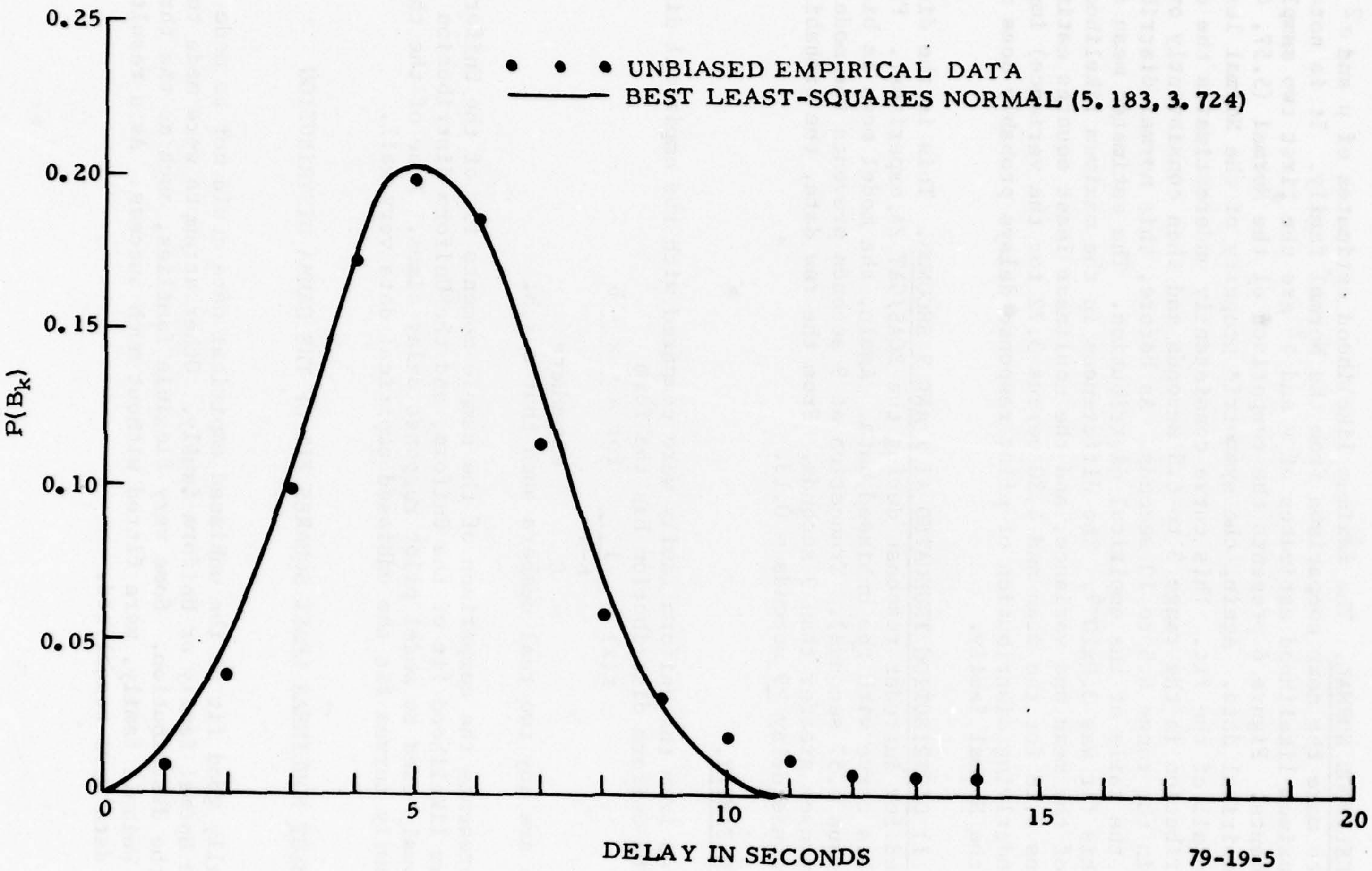


FIGURE 5. BEST NONLINEAR LEAST SQUARES NORMAL FIT

MAXIMUM LIKELIHOOD NORMAL. The maximum likelihood estimates of μ and σ^2 were used to make the next comparison from the Normal family. It is noted that the maximum likelihood estimates of μ and σ^2 are the first two sample central moments. Figure 6 presents the comparison of the Normal (5.57, 4.80) with the empirical data. Again, the symmetric property of the Normal leads to the downfall of the fit. This curve consistently underestimates the empirical distribution in the range 3 to 6.5 seconds and then consistently overestimates in the range 6.5 to 10 seconds. As before, this normal distribution cannot fit the tails of the empirical distribution. The estimated mean square error of this fit was 3.0×10^{-4}. The differences in the maximum likelihood estimates of the mean and variance, and the nonlinear least squares estimate (5.57 versus 5.18 for the mean and 4.80 versus 3.72 for the variance) imply that the underlying distribution of pilot response delays probably does not belong to the Normal family.

NORMAL (7, 1) DISTRIBUTION TRUNCATED AT 1 AND 9 SECONDS. This is the distribution modeled for intruder response during the BCAS/GAT 2A experiment. Figure 7 compares this curve with the unbiased data. Again, the model mean is biased (6.95 seconds versus 5.57 seconds). Truncation at 9 seconds prevents the modeling of any responses greater than 9 seconds. From the raw data, the probability that a response delay ≥ 9 seconds = 0.13.

THE UNIFORM FAMILY.

Three curves from the Uniform family were compared with the empirical distribution. The Uniform distribution has the form

$$f(x) = \frac{1}{b-a} \quad \text{for } a \leq x \leq b$$
$$= 0 \quad \text{elsewhere}$$

where a, b are any two real numbers such that $a < b$.

Figure 8 presents the comparison of the sample moments fit of the Uniform, the maximum likelihood fit of the Uniform, and the Uniform distribution that was previously used to model pilot response delay times. None of the three uniform family curves fit the unbiased empirical data very well.

BEST NONLINEAR LEAST SQUARES FIT OF THE GAMMA DISTRIBUTION

A universally good fit of the unbiased empirical data could not be made from either the Normal family or Uniform family. Other attempts were made to identify the distribution. Some very flexible families, such as the three-parameter Weibull family, were fitted without much success. As a result, the empirical data were reanalyzed.

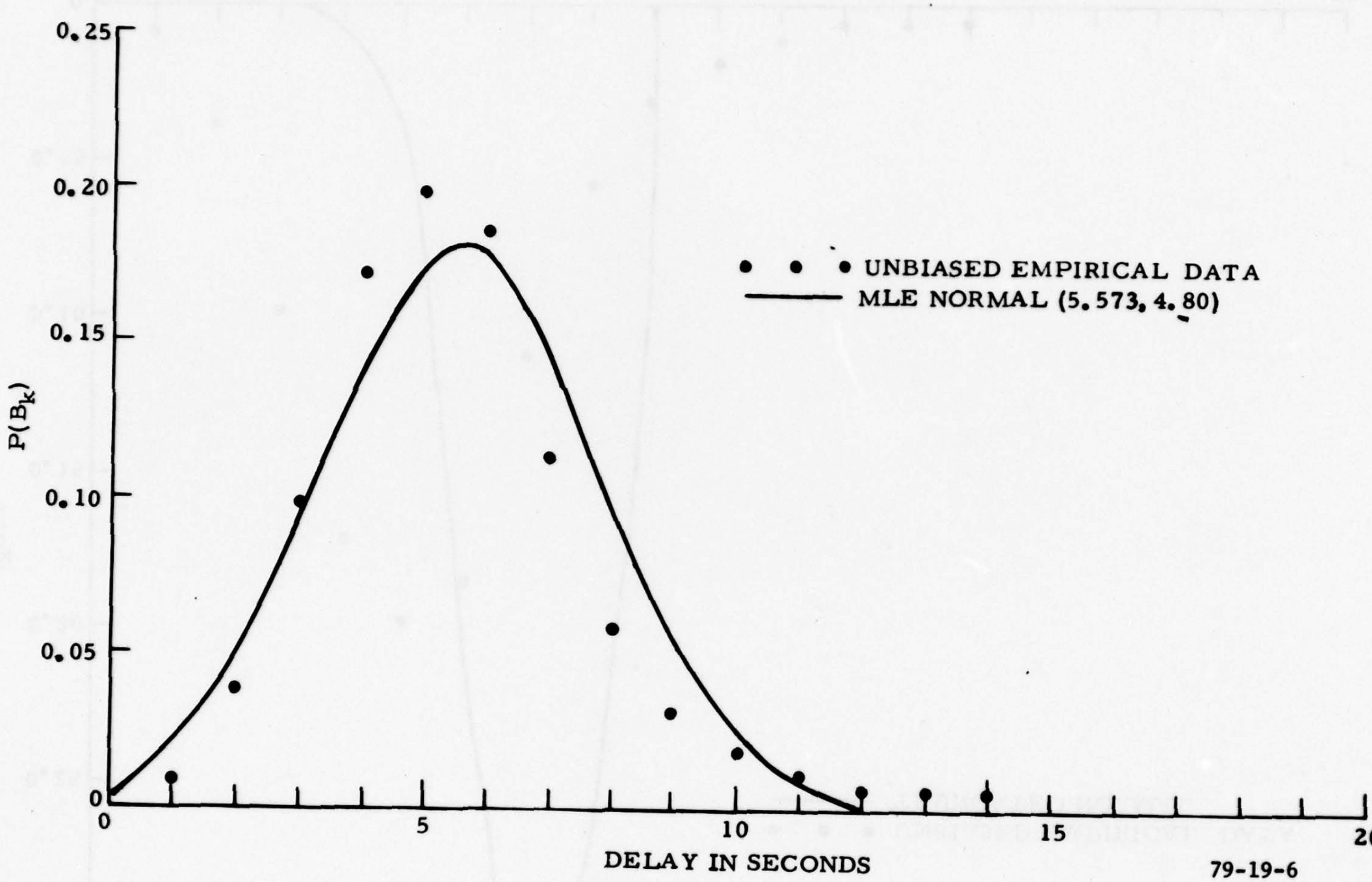


FIGURE 6. MAXIMUM LIKELIHOOD NORMAL FIT

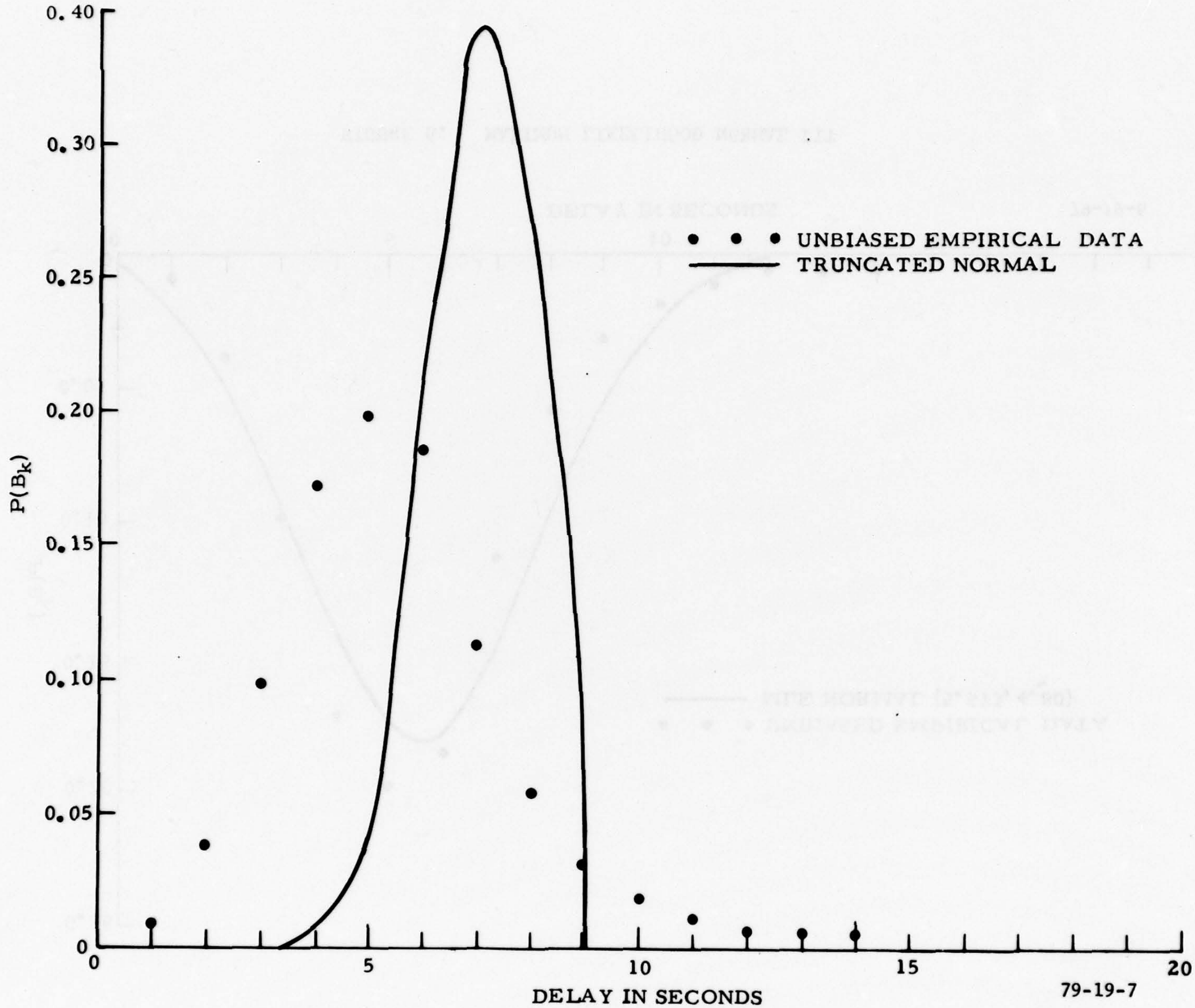


FIGURE 7. THE ATCSF TRUNCATED NORMAL FIT

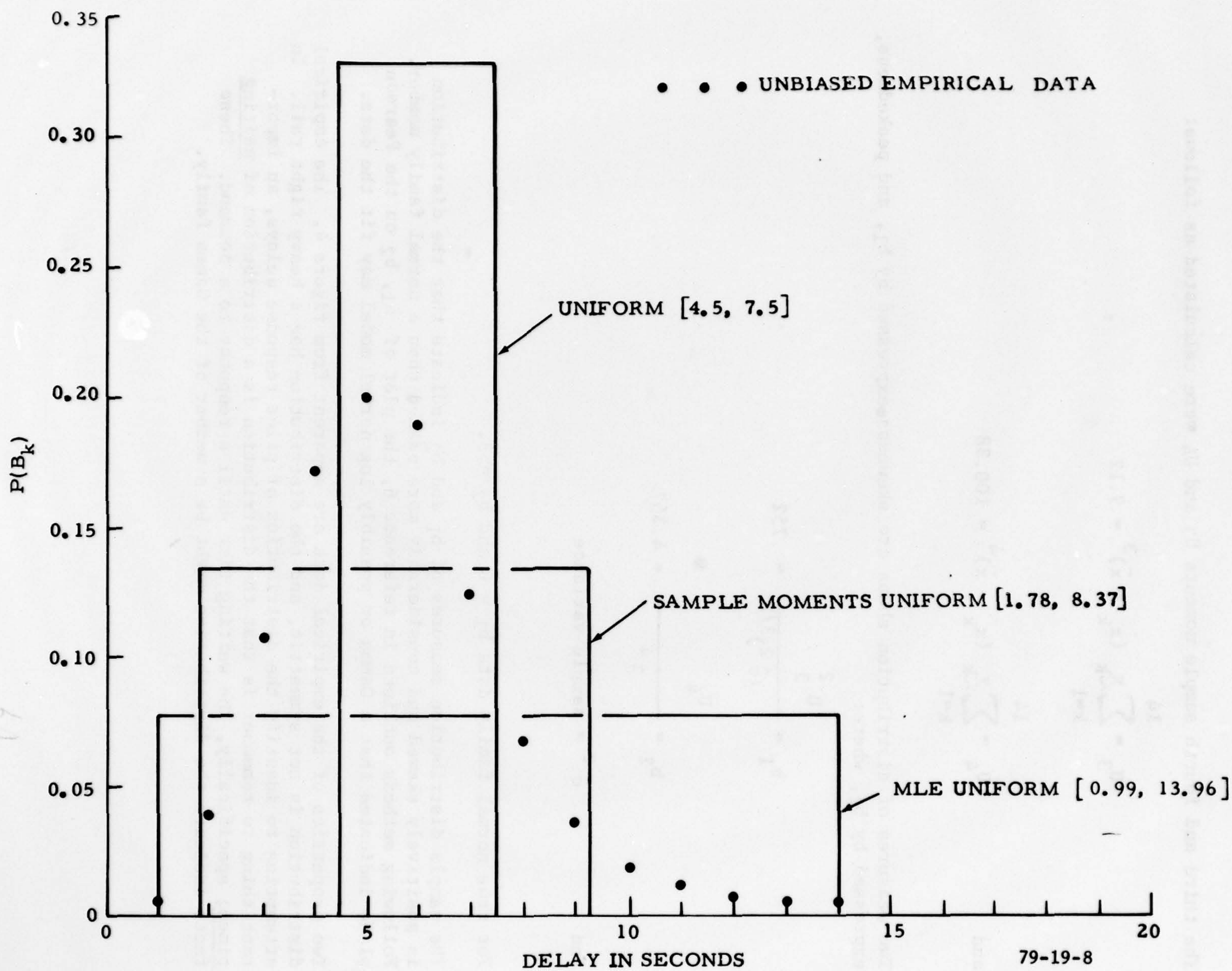


FIGURE 8. VARIOUS UNIFORM FAMILY FITS

The third and fourth sample moments U_3 and U_4 were calculated as follows:

$$U_3 = \sum_{k=1}^{14} \underset{\sim}{X}_k \, (x_k - \bar{x})^3 = 9.12$$

and

$$U_4 = \sum_{k=1}^{14} \underset{\sim}{X}_k \, (x_k - \bar{x})^4 = 100.38$$

Two measures of distribution shape are skewness expressed by b_1, and peakedness, expressed by b_2, where:

$$b_1 = \frac{U_3^2}{(\hat{\sigma}^2)^{3/2}} = .752$$

$$b_2 = \frac{U_4}{\hat{\sigma}^4} = 4.357$$

and $\hat{\sigma}^2$ = sample variance

For true normal family data $b_1 = 0$ and $b_2 = 3$.

The sample distribution measures of b_1 and b_2 indicate that the distribution is positively skewed and considerably more peaked than a normal family member. Following methods outlined in reference 6, the plot of b_1, b_2 on the Pearson plane indicates that a Gamma or possibly log normal model may fit the data.

Two properties of the empirical data are apparent from figure 4. The empirical distribution is not symmetric, and the distribution has a heavy right tail. In attempting to identify the distribution of pilot response delays, an important thing to remember is that the distribution is a distribution of <u>waiting times</u>; specifically, the waiting time until a response to a command. These facts indicate the distribution might be a member of the Gamma family.

THE GAMMA DISTRIBUTION.

The Gamma distribution is a two-parameter exponential family distribution with

$$f(x) = \frac{\lambda^r \; x^{r-1} \; e^{-\lambda x}}{\Gamma(r)} \qquad \text{for } 0 \leq x < \infty \qquad (1)$$

$$= 0 \text{ elsewhere}$$

where:

$\lambda > 0$ and is the scale parameter

$r > 0$ and is the shape parameter

Additionally, $\Gamma(r) = \int_0^\infty t^{r-1} \; e^{-t} \; dt$, $\Gamma(r) = (r-1)!$ for $r=1, 2, 3, \ldots$

is called the Gamma Function.

If $r = 1$ in (1), $f(x) = \lambda e^{-\lambda x}$ is the exponential distribution.
In queuing theory, this distribution plays a large role in modeling service distributions and waiting time distributions.

GENERATING MECHANISM.

Karlin (reference 7) presented a queuing theory model very analogous to the process that generates the pilot response delay times. The pilot is the single server in the queuing system. During his busy period, he is performing certain flight tasks, such as maintenance of heading, pitch, attitude, and airspeed. The onset of these tasks represent the "arrivals" in the queuing system.

The single server, the pilot, in performance of the flight tasks can be considered to be a very complex servo mechanism. Performance (servicing) of flight tasks is accomplished in r different stages, with the possibility of some tasks in different service stages being served simultaneously. A specific task may not need to pass through all r stages in order to complete service. The positive BCAS command is considered to be a flight task to be performed in the service queue. The positive BCAS command is not serviced until the pilot starts to make control movements that will cause the GAT 2A to begin to maneuver in the desired direction.

If the times required to service the arrivals in each service stage are all distributed exponentially with parameter λ, Karlin has shown that the waiting time (pilot response delay time) for a task to be serviced is distributed as a Gamma distribution with parameters r and λ. In the same reference, Karlin has extended the results to include any general distribution of arrivals. If the service times for each service stage are not identically distributed, McGill and Gibbon (reference 8) have shown that the response time distribution can be approximated by the generalized Gamma distribution. A schematic of the

possible service stages that a pilot might perform in responding to a command is presented in figure 9.

FITTING THE CURVE.

Since both parameters of the Gamma distribution are nonlinear, the BMD07R Nonlinear Least Squares program (reference 5) was used to provide estimates of the least squares best-fit Gamma distribution. The parameters are estimated using a stepwise Gauss-Newton iterative procedure. In addition to equation (1), the partial derivatives with respect to the parameters were needed. The partial derivatives are given by

$$\frac{\partial f(x;\lambda,r)}{\partial\lambda} = \frac{x^{r-1}e^{-\lambda x}}{\Gamma(r)} \cdot \left\{ r\lambda^{r-1} - \lambda \right\} \tag{2}$$

and

$$\frac{\partial f(x;\lambda,r)}{\partial r} = \frac{\lambda^{r}x^{r-1}e^{-\lambda x}}{\Gamma^{2}(r)} \left\{ \Gamma(r) \cdot \mathrm{Log}(\lambda r) - \Gamma'(r) \right\} . \tag{3}$$

The use of the stepwise Gauss-Newton iterative procedure requires the initial estimates of the parameters be provided. To insure rapid convergence in the procedures, the maximum likelihood estimates were used as initial values. The maximum likelihood estimates r and λ are the solution to the following system of equations.

$$\frac{rn}{\lambda} - \sum_{i=1}^{n} \underset{\sim}{X}_i = 0 \qquad n = 253 \tag{4}$$

and

$$n\mathrm{Log}(\lambda) + \sum_{i=1}^{n} \mathrm{Log}(\underset{\sim}{X}_i) - \frac{n \cdot \Gamma'(r)}{\Gamma(r)} = 0 \tag{5}$$

From reference 9,

$$\Gamma(r)^{-1} = r\exp(\gamma \cdot r) \left\{ \prod_{i=1}^{\infty} (1 + r/i)\exp(-r/i) \right\} \tag{6}$$

where γ = Euler's Constant = .57721. . .

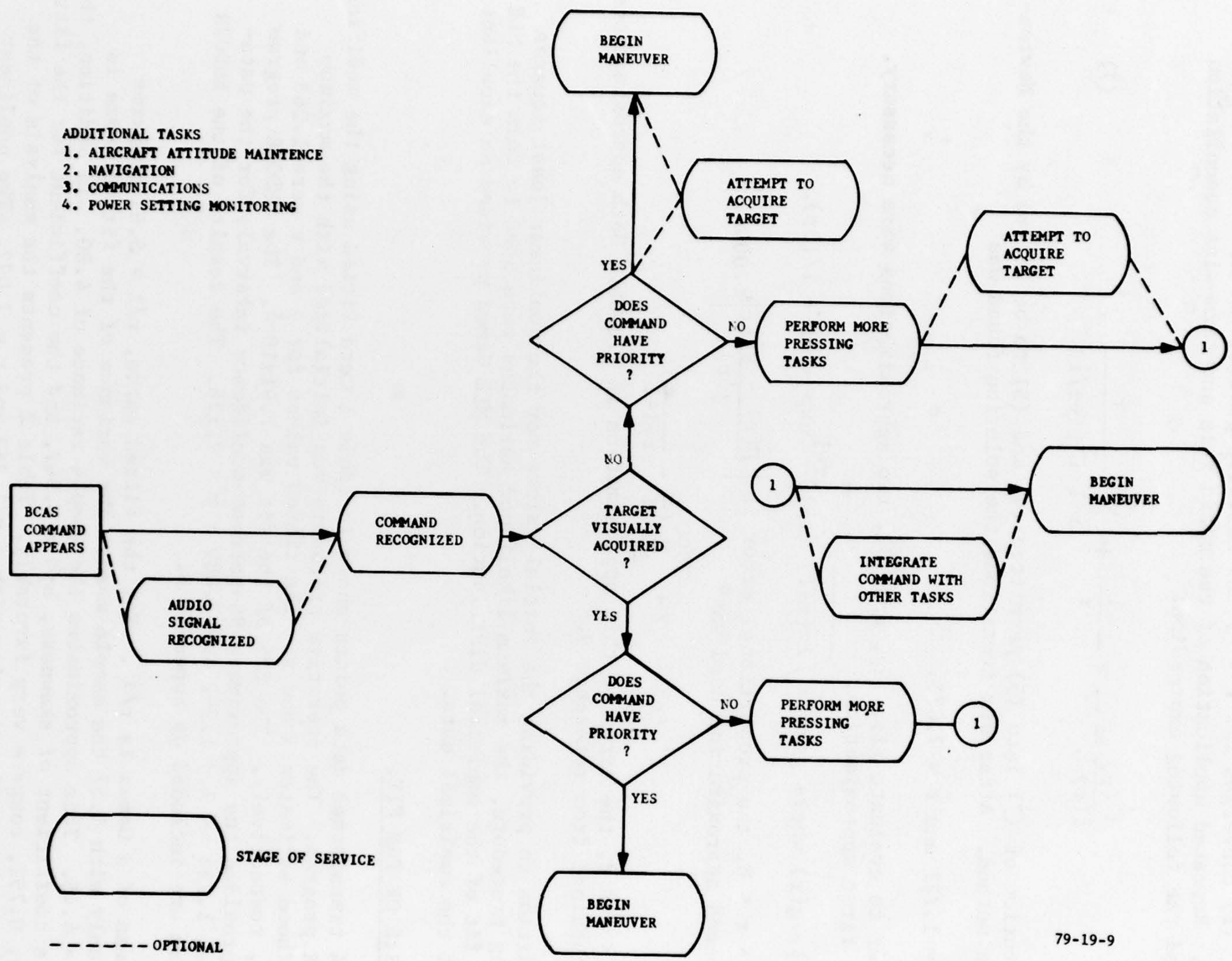


FIGURE 9. SCHEMATIC OF POSSIBLE SERVICING STAGES

Since Γ (r) converges for $r < \infty$, the infinite product in (6) can be differentiated. Repeated application of the chain rule and term-wise cancellation obtained the following expression:

$$\frac{\Gamma'(r)}{\Gamma(r)} = -\gamma - \frac{1}{r} + \sum_{i=1}^{\infty} \frac{r}{i^2(1+r/i)} \quad . \tag{7}$$

Substitution of (7) into (5) permitted (4) and (5) to be solved by the Newton-Raphson method. After six iterations the solution found was

$\hat{\lambda} = 1.271$ and $\hat{r} = 7.073$.

In order to evaluate (1), (2), and (3), two approximations were necessary. In the first approximation,

$$\Gamma(r) \sim g(r) \text{ where } g(r) = \sqrt{2\pi(r-1)}\ (r-1)^{r-1} \exp(-r+1 + 1/10r).$$

For $7 < r < 8$, the proportional error is $\left| \frac{\Gamma(r) - g(r)}{g(r)} \right| < .0008$.
The second approximation used was

$$\Gamma'(r) \sim -\gamma + \sum_{i=1}^{3000} \left(\frac{1}{i} - \frac{1}{r+i-1}\right).$$

For $7 < r < 8$, the error of this approximation is $\leq .0006$. Both approximations were obtained from reference 9.

In addition to providing the initial values for the nonlinear least squares fitting procedure, the maximum likelihood estimates were used to form the MLE Gamma fit of the empirical distribution. The MLE Gamma provided an excellent fit of the empirical data.

ANALYSIS OF THE FIT.

The 14 transformed data points shown in table 1 were fitted using the modified BMDO7R program. The iterative procedure was initialized with the maximum likelihood estimates λ and r. The fitted values for λ and r were 1.263 and 7.092, respectively. The EMS of the fit was 7.91×10^{-5}. The BMDO7R program also provided the approximate 95-percent confidence intervals for the parameters, $1.234 < \lambda < 1.292$ and $7.009 < r < 7.176$. The results of the BMDO7R program are included as appendix B.

The mean of a Gamma is r/λ . For the fitted curve, $r/\lambda = 5.61$ compares favorably with 5.57 the sample mean. The variance of the fitted Gamma is $r/\lambda^2 = 4.45$. This approximates the sample variance of 4.80. In addition, the sample coefficient of skewness, $b_1^{1/2} = 0.867$, and the coefficient for the fitted curve, 0.752, compare very favorably. Table 2 presents the analysis of the fit of the Gamma distribution with $\lambda = 1.263$ and $r = 7.092$. The nonlinear least squares best fit Gamma is graphically compared with the empirical data in figure 10.

TABLE 2. ANALYSIS OF GAMMA FIT

Delay Seconds	Experimental Proportion	Predicted Proportion	Error
1	0.0073	0.0017	0.0056
2	0.0437	0.0334	0.0103
3	0.1056	0.1116	-0.0060
4	0.1747	0.1821	-0.0074
5	0.1991	0.2006	-0.0015
6	0.1912	0.1724	0.0188
7	0.1217	0.1246	-0.0029
8	0.0695	0.0795	-0.0100
9	0.0315	0.0460	-0.0145
10	0.0225	0.0248	-0.0023
11	0.0140	0.0125	0.0015
12	0.0093	0.0060	0.0033
13	0.0050	0.0028	0.0022
14	0.0050	0.0012	0.0038

COMPARISONS.

The Normal fits to the empirical data were all affected by the symmetric property of the Normal distribution. The symmetric property caused a biased estimate of the mean for the best nonlinear least squares Normal fit.

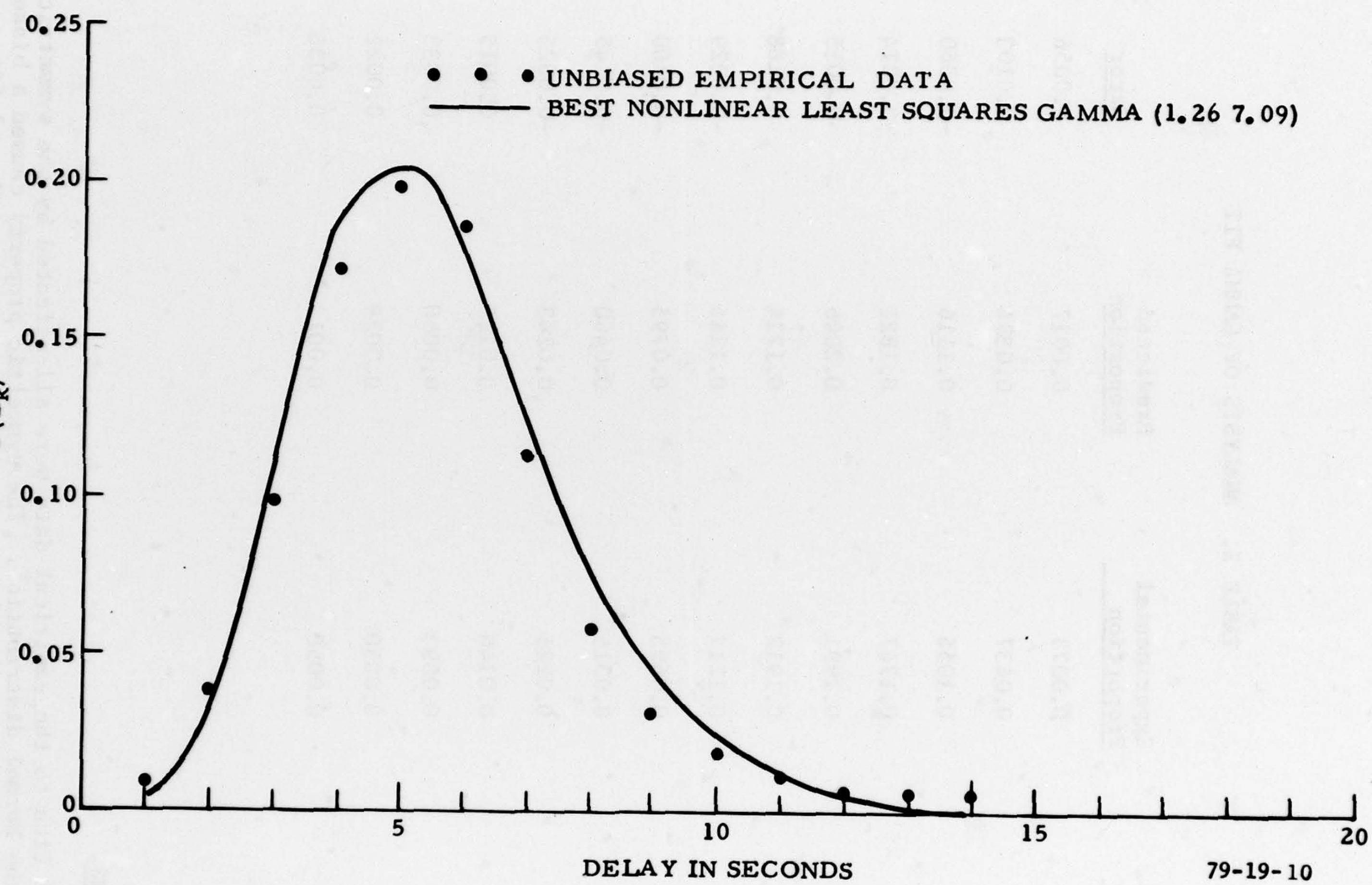


FIGURE 10. BEST NONLINEAR LEAST SQUARES GAMMA FIT

The differences in the estimates of the mean and variance for the best nonlinear least squares estimates and the maximum likelihood estimates (MLE) tend to invalidate the use of a Normal distribution in modeling the pilot response delays. The minimum estimated mean square error for any normal fit of the empirical data is 8.4×10^{-5}.

Table 3 compares the bias and standard deviation of each model fitted with the unbiased empirical distribution data. Although the error mean square for the nonlinear least squares normal fit almost matches the nonlinear least squares Gamma, its mean is highly biased (-0.39 seconds). It is highly unlikely that this normal distribution (the one that fits the data the best) is the correct model for pilot response delays. If in fact the underlying distribution was Normal with a mean of 5.18 seconds, a T test can be made to compare this mean with the sampled mean of 5.57 seconds.

$$\text{Let } H_o: \mu_o = 5.18$$

$$H_a: \mu_a \neq \mu_o$$

then

$$T = \frac{(\bar{x} - \mu_o)\sqrt{n}}{s} = 2.83$$

From T tables with 252 degrees of freedom, the probability that T > 2.56 is .005.

CONCLUSIONS

The Gamma distribution fitted by the nonlinear least square method provided the smallest estimated mean square error, 7.091×10^{-5}. The mean r/λ and the variance r/λ^2 of the fitted Gamma closely approximated the sample mean and variance. The selection of the Gamma as the model for pilot response delays is also supported intuitively based on a comparison of the generating mechanism to a queuing theory model.

The results of this analysis were based on data that resulted from experimentation using general aviation pilots as subjects. The simulator modeled a medium twin-engine piston-powered aircraft. The response model for other pilot groups flying different equipment might not be the same.

Analysis of the effect that the use of the incorrect pilot response model at NAFEC had on previous BCAS studies was made. No major effect was detected. However, the two distributions previously used do not fit the emperical data as well as the Gamma distribution. These distributions did not have the flexibility that the Gamma family has to model both the significant portion of early responses (<4 second delays) and the late responses (>8 second delays). Figure 11 shows the differences in the cumulative probability functions for the unbiased empirical data, the fitted Gamma, the Uniform (4.5, 7.5), and the

TABLE 3. COMPARISON OF FITS

Distribution Family	Parameters		EMS	Mean	Bias	Stand Deviation
Unbiased Empirical				5.573		2.191
Normal Family						
ATCSF Model	7.0	1.0	1.2×10^{-2}	6.954	1.381	0.885
MLE Normal	5.573	2.191	3.0×10^{-4}	5.573		2.191
Least Squares Normal	5.183	1.928	8.4×10^{-5}	5.183	-.390	1.928
Uniform Family	a	b		$(a+b)/2$		$(b-a)/12$
Previous Model	4.5	7.5	1.0×10^{-2}	6.000	0.427	0.866
MLE Uniform	0.990	13.960	5.4×10^{-3}	7.475	1.902	3.744
Sample Moments	1.778	9.368	2.6×10^{-3}	5.573		2.191
Gamma Family	λ	r		r/λ		r/λ^2
MLE Gamma	1.272	7.073	8.3×10^{-5}	5.561	-0.012	2.091
Nonlinear Least Square Gamma	1.263	7.092	7.9×10^{-5}	5.615	0.042	2.109

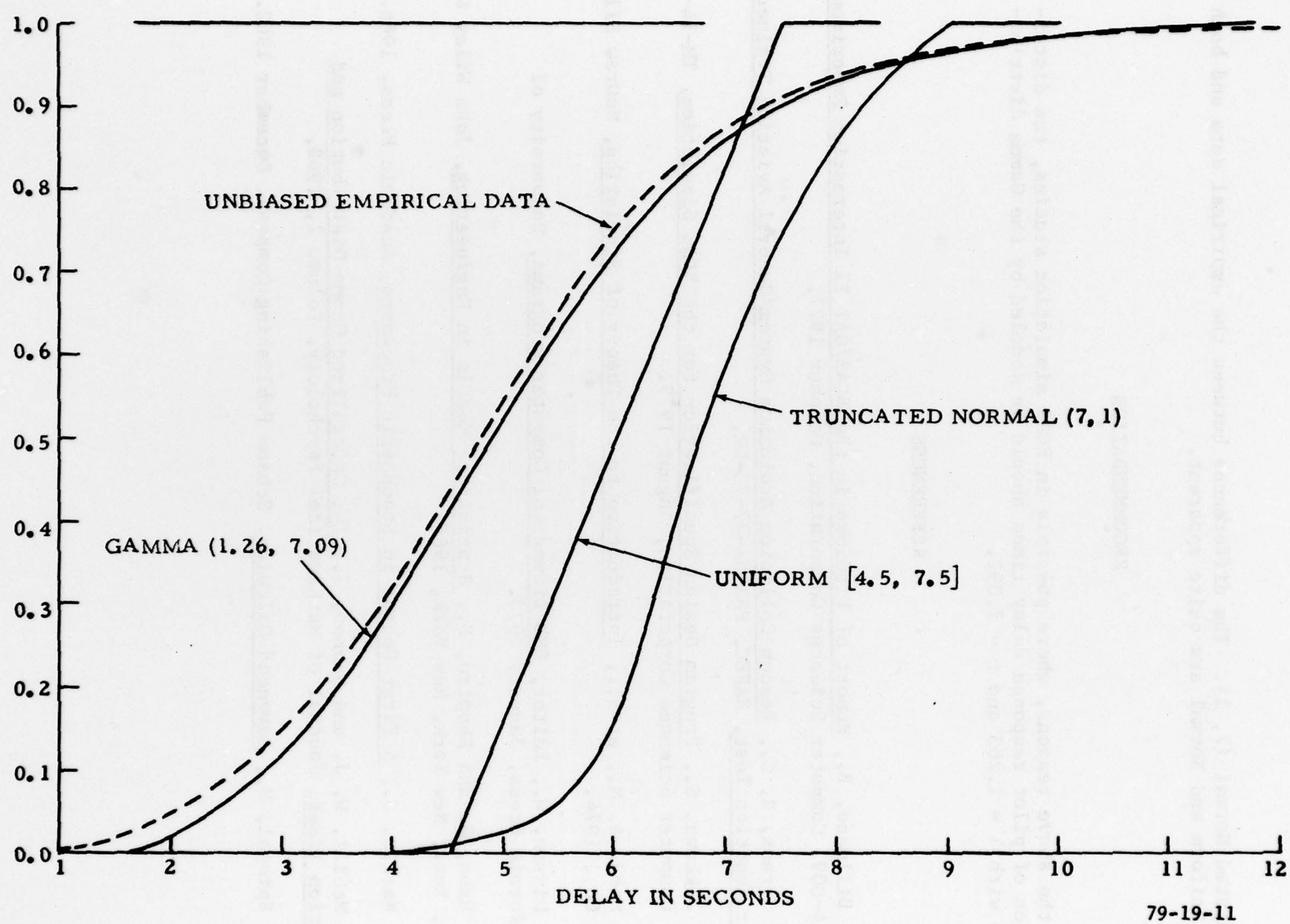


FIGURE 11. CUMULATIVE PROBABILITY DISTRIBUTIONS

truncated Normal (7, 1). The differences between the empirical data and both the Uniform and Normal are quite apparent.

RECOMMENDATION

For the above reasons, where possible in BCAS simulation studies, the distribution of pilot response delay times should be modeled by the Gamma distribution with $\lambda = 1.263$ and $r = 7.092$.

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APPENDIX A - POOLING STRATEGY

Data collected during experimentation reflected pilot response delays to four types of commands (climb, descend, turn left, and turn right) and two BCAS sensitivity levels (high sensitivity or early warnings and low sensitivity or late warnings). The sample pilot response delay population variances and all sample sizes are depicted in table A-1.

TABLE A-1 - SAMPLE VARIANCES

COMMAND TYPE	SENSITIVITY	NUMBER IN CELL	$\hat{\sigma}^2$
CLIMB	HIGH	42	3.88
CLIMB	LOW	24	2.72
DESCENT	HIGH	34	4.97
DESCENT	LOW	26	5.76
RIGHT TURN	HIGH	35	5.43
RIGHT TURN	LOW	24	3.20
LEFT TURN	HIGH	44	4.00
LEFT TURN	LOW	24	5.24

Prior to beginning any curve-fitting procedures, tests were performed to support pooling of data. Mood, et al. (reference 4) described the following approximate chi-square test for the equality of several variances.

If $\hat{\sigma}_j^2$ = sample population variance for population j

n_j = sample size for population j $j = 1, 2, \ldots, k$

$$N = \sum_{j=1}^{k} n_j$$

and

$$\lambda = \frac{\prod_{j=1}^{k} \left(\hat{\sigma}_j^2 \right)^{n_j/2}}{\left(\sum_{j=1}^{k} n_j \hat{\sigma}_j^2 \Big/ \sum_{j=1}^{k} n_j \right)}$$

then the statistic $-2 \text{ Log } (\lambda)$ is approximately chi-squared with (k-1) degrees of freedom.

If H_o : is $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

and H_a : is $\sigma_i^2 \neq \sigma_j^2$ for some $i \neq j$

then the $p = .05$ test would reject H_o when $-2\text{Log}(\lambda) > \chi^2_{(k-1)}(.95)$.

From our data $-2\text{Log}(\lambda) = 3.2$ and $-2\text{Log}(\lambda) < \chi^2(7) = 14.1$. Hence, the hypothesis that the sample variances are all equal cannot be rejected, and the pooling of data is supported.

The pooling resulted in a sample variance estimate of 4.80 seconds squared and a mean of 5.57 seconds.

APPENDIX B
BMD RESULTS

BMD07R - NONLINEAR LEAST SQUARES - REVISED JAN. 1, 1973

HEALTH SCIENCES COMPUTING FACULTY, UCLA

PROBLEM CODE		GAMMA
NUMBER OF VARIABLES		2
INDEX OF DEPENDENT VARIABLE		2
INDEX OF WEIGHTING VARIABLE		0
NUMBER OF CASES		14
NUMBER OF PARAMETERS		2
TOLERANCE		.00100
EPSILON		.00010
MAXIMUM NUMBER OF ITERATIONS		100
MINIMA	3.00E-01	1.800E00
MAXIMA	5.000E00	3.000E01

ITERATION	EMS	PARAMETERS	
		λ	r
0	8.2646E-05	1.2714	7.0729
1	8.0230E-05	1.2594	7.1048
2	7.9150E-05	1.2641	7.0897
3	7.9139E-05	1.2630	7.0922
4	7.9139E-05	1.2630	7.0922
5	7.9139E-05	1.2630	7.0922
6	7.9139E-05	1.2630	7.0922
7	7.9139E-05	1.2630	7.0922

ASYMPTOTIC STANDARD DEVIATIONS OF THE PARAMETERS

2.9185E-02 8.4037E-02